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Dynamics and Control of the Activated Sludge Wastewater Process

The dynamics of the activated sludge process are governed under certain conditions by interactions between the reactor and settler through sludge recycle. Settler underloading can lead to extremely sluggish system response, indicating that effective sludge height regulation is an important control objective. This objective may be in conflict with the need to maintain small variations in reactor solids concentration. An effective compromise can be achieved by using ratio control on both sludge recycle and settler underflow. This control policy does not require sludge storage.

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SCOPE

The activated sludge process is the most widespread method of secondary wastewater treatment. The process consists of a continuous flow biochemical reactor followed by a continuous settler, with recycle of active biological sludge to the reactor. The process typically experiences large changes in feed flow rate and dissolved organics over time scales that are comparable both to reactor and settler residence times and to the reciprocal of the pseudo first-order rate constant for reaction of organics. Thus, the process will rarely operate at steady state and may require controls for good performance.

The settler response can be important in determining the overall process dynamics because of the close coupling between reactor and settler. Previous control studies have used inadequate models of the settler that cannot account for possible adverse interactions between the units and resulting deterioration of performance. This paper describes the results of a simulation study of the dynamics and control of the activated sludge process, using a recently developed model of the dynamics of continuous sedimentation that accounts for several distinct modes of operation. Interactions between reactor and settler dynamics are particularly considered.

CONCLUSIONS AND SIGNIFICANCE

The simulated dynamical response deteriorates significantly when the settler is underloaded, because the buffering action of the sludge blanket is lost and reactor disturbances are positively reinforced by the recycle loop. This suggests that a primary goal of a control system should be to maintain the sludge level in the settler. Sludge blanket control can lead to large swings in reactor solids concentration, however, which is also undesirable. Conversely, a control system designed to regulate only the reactor can cause the settler to become underloaded or overloaded.

An effective compromise between these two conflicting goals can be achieved using an elementary strategy of hydraulic control, in which the rates of settler underflow and recycle to the reactor are kept at constant ratios to the feed flow rate. This strategy diminishes variations in both reactor solids concentration and settler sludge blanket height while requiring only flow measurements. Unlike some other proposed control strategies, no storage of sludge is required.

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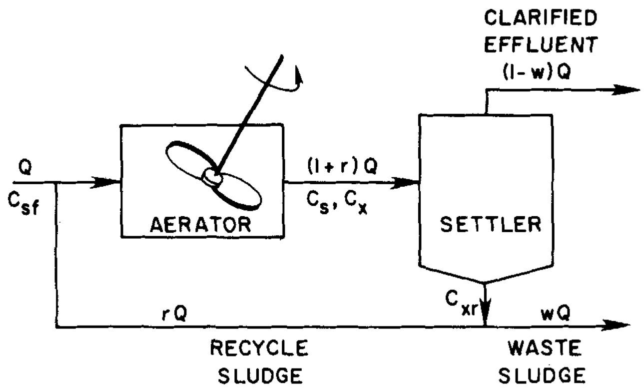

Fig. 1. Schematic of activated sludge process.

The activated sludge process, shown schematically in Figure 1, is the most widespread method of secondary wastewater treatment. The aerator is a reactor in which suspended organisms biochemically attack dissolved organics (the substrate). The suspended organisms are separated in the settler, with clarified effluent taken out overhead. A small portion of the concentrated suspension is wasted to maintain a bounded organism level in the system, and the remainder is recycled to the aerator.

Although the reactor-settler system is conventionally designed for steady state operation, the process in fact experiences large changes in flow rate and substrate feed concentration over a 24 hr cycle. The dynamical response of the biochemical reactor alone is reasonably well understood; the absence until recently of adequate dynamic models for continuous settlers has precluded detailed study of the dynamics of the integrated recycle process. The settler dynamics can dominate the process response, so control studies based on inadequate settler models may give misleading or incorrect results. The status of dynamics and control studies for the activated sludge process has recently been comprehensively reviewed by Olsson (1976).

We report here on results of a simulation study of activated sludge dynamics and control using a settler model which accounts for the possibility of different regimes of settler performance. When the settler is underloaded, the response time of the overall unit may be increased by an order of magnitude over the response times of the individual units. The need to keep the settler from becoming underloaded and to keep the reactor solids concentration from varying widely could lead to conflicting control needs under some conditions.

REACTOR MODEL

The aerator is taken to be a single well-stirred tank; the generalization to other reactor configurations is straightforward, and simulations for a multistage reactor show the same qualitative performance as that reported here. All substrate species are lumped into a single pseudospecies known as biochemical oxygen demand (BOD), with a concentration denoted by C_s. All organisms are lumped into a single pseudospecies with a concentration denoted by C_x. The flow rates into and out of the reactor are assumed to be equal, in which case the reactor equations are

$$V\frac{dC_s}{dt} = QC_{sf} + rQC_{sr} - Q(1+r)C_s - \frac{1}{Y}VR \tag{1}$$

$$V\frac{dC_x}{dt} = rQC_{xr} - Q(1+r)C_x + VR \tag{2}$$

Subscript f denotes the feed and r the recycle stream. Y is a stoichiometric "yield factor" relating substrate usage to organism growth.

The growth rate is usually taken to follow the Monod equation

$$R = \mu C_x \tag{3}$$

$$\mu = \frac{\hat{\mu} C_s}{k + C_s} \tag{4}$$

The rate depends on the dissolved oxygen concentration only up to a critical oxygen level, after which oxygen dependence is slight. We shall assume here that the oxygen level is always above the critical value. Typical values of the rate parameters, which are used in all simulations reported here, are $\hat{\mu} = 0.5$ hr^{-1}, $Y = 0.5$, and $k = 0.05$ kg/m^3.

There is some evidence to suggest that growth kinetics are time dependent and follow a first-order response (Young et al., 1970; Young and Bungay, 1973). The appropriate form which leads to the Monod equation at steady state is

$$\tau\frac{d\mu}{dt} = \frac{\hat{\mu} C_s}{k + C_s} - \mu \tag{5}$$

The time constant τ is thought to be of the order of several hours; Blanch (1976) suggests that τ will be of the order of the doubling time at the previous steady state, which is the reactor residence time. Some simulations incorporating the dynamic lag in the growth rate have been carried out. The lag can induce large oscillations in substrate concentration for a system with sludge recycle, but solids concentration levels are unaffected. Thus, the growth rate dynamics have not been included in the dynamics and control studies reported here, where solids concentrations are the variables of greatest interest.

SETTLER MODEL

The settler model has been described in detail elsewhere (Attir et al., 1977). It is a generalization of the model developed by Chi and Howell (Chi, 1974), which is similar to the model of Tracy and Keinath (1974). The fundamental idea follows from the design equations

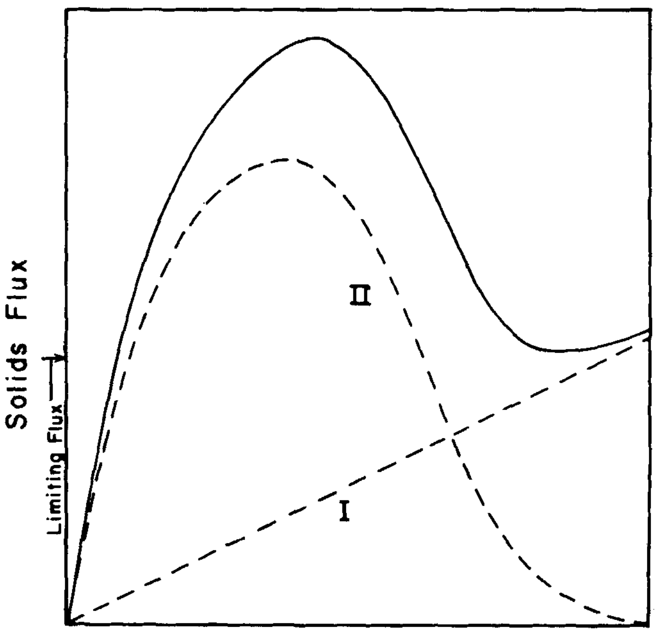

Fig. 2. Typical flux curve for settler operation. Dashed lines I and II are convective and gravity fluxes, respectively; the solid line is the total flux, equal to the sum of the two.

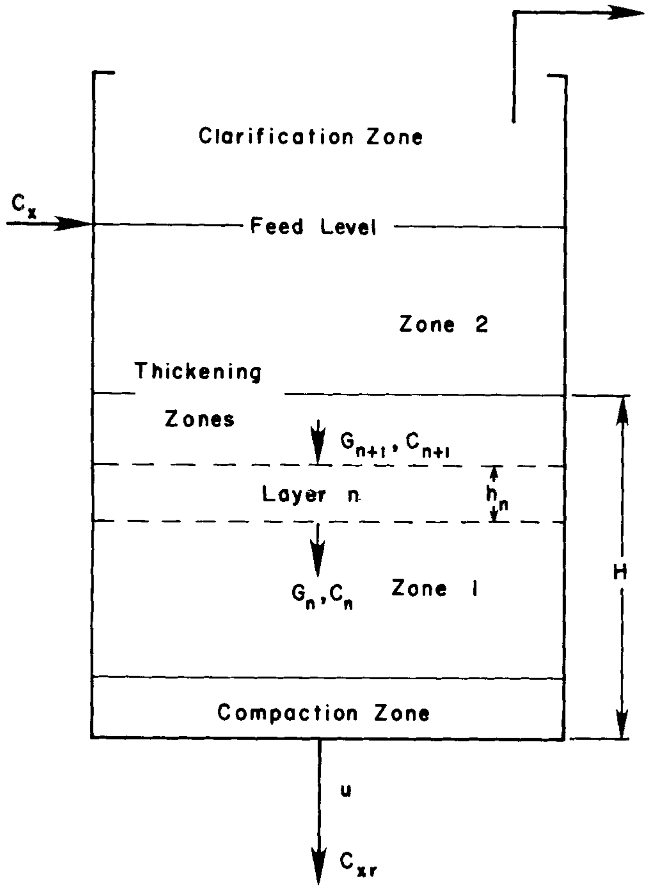

Fig. 3. Schematic of Chi-Howell settler model, after Chi (1974).

for steady sedimentation. At steady state, the solids flux G at any level in the settler can be written

$$G = C[u + v(C)] \tag{6}$$

Here, u is the velocity at which the underflow is removed ($[r + w]Q$/settler area), so Cu represents a convective flux. $v(C)$ is the velocity at which a suspension of concentration C settles by gravity, so $Cv(C)$ is the gravity settling flux relative to the convective motion. A typical flux curve is shown in Figure 2; dashed lines I and II are the convective and gravity fluxes, respectively, and the total flux is the solid line, which is the sum of the two.

The minimum in the flux curve, known as the limiting flux, plays a central role in settler design because it is believed that fluxes above the limiting flux cannot be sustained in the system (Dick, 1970). Since the flux must be the same at all points in the settler at steady state, it follows that the only concentrations possible in the settler are those at the same horizontal level in Figure 2. Thus, only discrete layers with constant composition are possible.

The Chi-Howell model is shown schematically in Figure 3. The model divides the settler into a constant volume clarification zone at the top and up to three sludge thickening zones, as follows:

1. A constant volume compaction zone, with solids concentration equal to the underflow concentration.

2. Zone 1, in which each concentration layer corresponds to the limiting concentration at some underflow velocity which the system has experienced in the past. These concentrations are time dependent to account for time variations in the solids flux to the thickening section. The interface between zones 1 and 2 is the height H of the thick blanket of sludge.

The mass balance equations for the n^{th} layer are

$$\frac{dh_n}{dt} = \delta_{n-1} - \delta_n \tag{7}$$

$$\frac{dC_nh_n}{dt} = (G_{n+1} - C_{n+1}\delta_n) - (G_n - C_n\delta_{n-1}) \tag{8}$$

δ_n is the velocity of motion of a solids concentration discontinuity:

$$\delta_n = \frac{G_{n+1} - G_n}{C_{n+1} - C_n} \tag{9}$$

A layer is removed whenever $h_n \leq 0$. A new concentration layer may form when either the underflow rate or the feed flux is changed. In the former case, a layer of infinitesimal height is created somewhere in zone 1 with a concentration equal to the limiting concentration corresponding to the particular underflow. The layer will grow in height as long as the solids flux entering the layer exceeds the limiting flux at that underflow rate; otherwise, the layer will not develop. A new layer will form at the top of zone 2 if the solids flux to the thickening section is less than the flux in the uppermost layer of zone 2. Momentum transport is ignored except for a case in which an adverse concentration gradient develops; that is, $C_n > C_m$, $n < m$. This is known to be an unstable configuration in which vertical circulations are induced, and in such a case the layers are assumed to mix instantaneously.

Equations (7), (8), and (9) represent an approximation to a hyperbolic partial differential equation, which can be solved for some limiting cases by the method of characteristics (Petty, 1975). Layered solutions may sometimes violate a condition equivalent to the second law of thermodynamics. In that case, the proper solution requires a continuous variation of concentration with height in this region (a rarefaction wave), but for simulation purposes, a series of discrete layers which span the concentration range is inserted. Details are described in Attir et al. (1977).

RECYCLE RATIO CONTROL

A primary goal of any control strategy for the activated sludge process must be to prevent large excursions in solids concentration in the reactor. An obvious way to

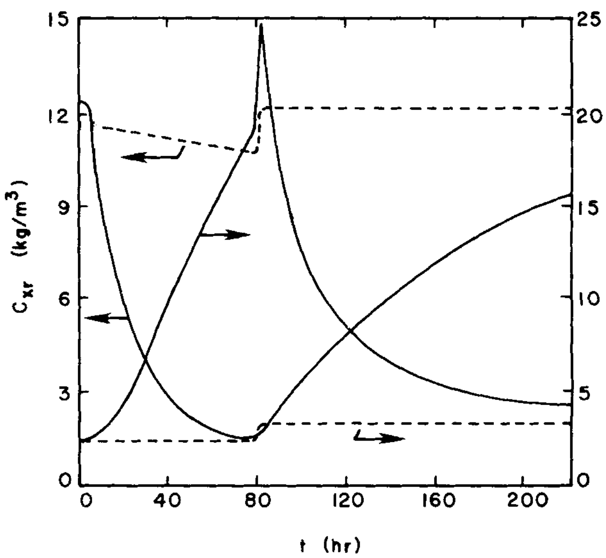

Fig. 4. Simulation of response to a 25% step decrease in flow rate at $t = 2$ hr, followed by a step increase to the original flow rate at $t = 80$ hr; the solid lines are recycle ratio control, and the dashed lines are constant recycle rate.

Table 1. Parameters for Simulations with Two Step Changes in Feed Rate

Area of settler	$= 80.2\ m^2$
Height of settler	$= 4\ m$
Feed height of settler	$= 3.25\ m$
Height of bottom layer	$= 0.1\ m$
Reactor volume	$= 800\ m^3$
Feed flowrate at $t = 0$	$= 200\ m^3/hr$
Recycle flow rate at $t = 0$	$= 40\ m^3/hr$
Underflow rate	$= 41.59\ m^3/hr$
Feed substrate conc.	$= 0.2\ kg/m^3$
Sludge blanket height at $t = 0$	$= 0.6\ m$

Table 2. Parameters for Simulations with 24 hr Cyclic Inputs

Area of settler	$= 103\ m^2$
Height of settler	$= 3\ m$
Feed height of settler	$= 2.25\ m$
Height of bottom layer	$= 0.1\ m$
Reactor volume	$= 822\ m^3$
Average feed flow rate	$= 137.8\ m^3/hr$
Average recycle rate	$= 34.45\ m^3/hr$
Average underflow rate	$= 34.65\ m^3/hr$
Sludge blanket height at $t = 0$	$= 1.1\ m$

accomplish this goal is to use recycle ratio control, in which the recycle ratio r is maintained at a fixed value. Recycle ratio control will serve as our reference point. If the underflow velocity u is maintained constant in order to prevent upsets in the settler, then a strict recycle ratio control policy will usually not be possible unless stored sludge is available for recycle ($w < 0$).

DYNAMICS WITH UNDERLOADED SETTLER

The simulation results shown in Figure 4 illustrate the influence of a badly underloaded settler on the dynamics of the integrated process. System parameters are given in Table 1. The reactor residence time is 3.3 hr, and the solids residence time in the settler is less than 6 hr; 6 hr is the time required for convective flow from the feed to the bottom of the settler. This low level was chosen to illustrate a point, but a low sludge level may be desirable to minimize solids residence time and to avoid organism death in the settler.

At time $t = 2$ hr there is a 25% step reduction in the flow rate to 150 m^3/hr; at $t = 80$ hr there is a step increase to the original 200 m^3/hr. The solid lines in Figure 4 show the response when the system is maintained under recycle ratio control, with $r = 0.2$. The sludge level falls rapidly, and after a short time the settler is underloaded; that is, there is no limiting layer in the settler. The effect on the dynamics is dramatic; steady state has not been reached 120 hr after the step increase in flow rate.

The dashed lines in Figure 4 show the same case with a single change. Here, the recycle flow rate was maintained at the steady state value at all times. The effect is to recycle slightly more solids and to prevent serious underloading in the settler. The approach to steady state following the step increase is rapid, despite a difference of less than 6% in reactor residence time in the two cases.

The essential difference between these two cases is the existence of a limiting layer in the settler. Such a layer acts as a buffer and allows the settler underflow concentration to remain relatively constant in time. This prevents reactor disturbances from being fed back and amplified. It is evident from this simulation result that a primary goal of a control strategy must be to prevent settler underloading because of the possible adverse effect on dynamics and the ability of the system to recover from upsets. The presence of a limiting layer effectively uncouples the reactor dynamics from those of the settler, eliminating the unfavorable effect of recycle on system dynamics first noted by Gilliland et al. (1964).

TWENTY-FOUR-HOUR CYCLE

Possible conflicts in the goals of preventing wide swings in reactor solids concentration and in preventing settler underloading are illustrated by considering a typical 24 hr cycle. Figure 5 shows flow rate and feed substrate concentration over a 24 hr period; the data are a discretization of time series used by Harris (1976). System parameters are recorded in Table 2. The design variables were based on the average flow rate and concentration, and the initial state for all simulations is steady state at this average.

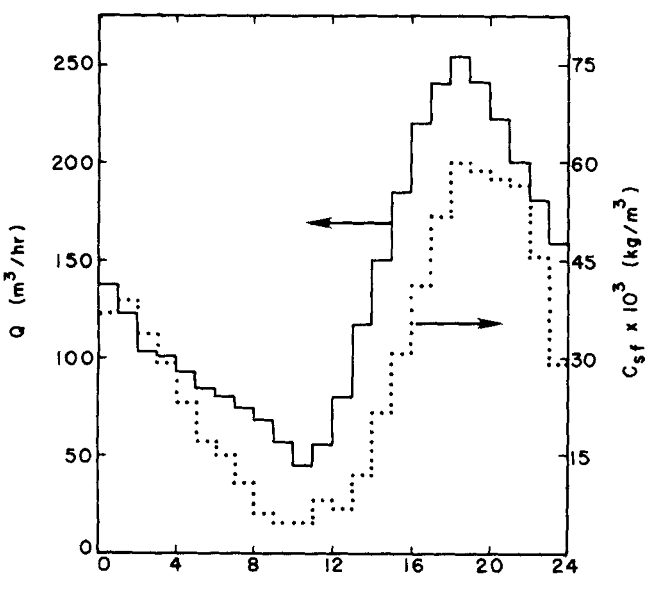

Fig. 5. Twenty-four-hour variations in feed flow rate and feed substrate concentration, after Harris (1976).

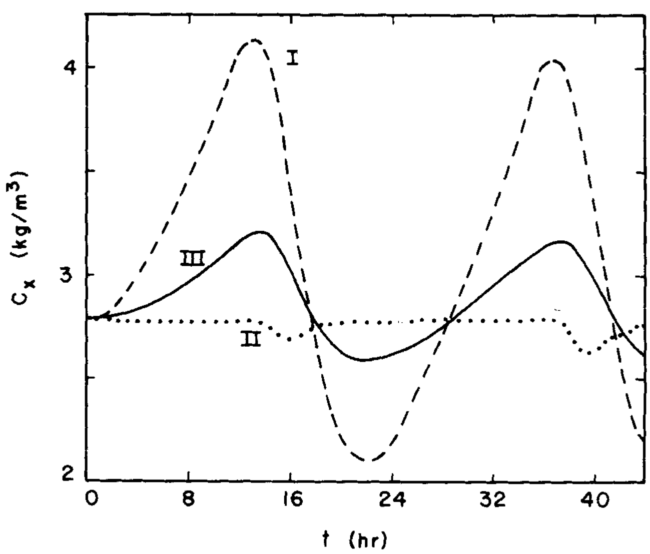

Fig. 6. Reactor solids concentration for input forcing in Figure 5. Line I, constant recycle and underflow rates. Line II, ratio recycle control and constant underflow rate. Line III, ratio control on recycle and underflow.

The response of the reactor solids concentration for operation with constant underflow and recycle flow rates is shown as line I in Figure 6. Underloading is avoided for the initial sludge height chosen, and as a result the buffering capacity of the settler is retained and the underflow solids concentration is constant. Thus, no reactor fluctuations are propagated through the settler and returned to the reactor. There is a large variation in solids concentration in the reactor, however.

Line II in Figure 6 shows the response under ratio control. The variation in reactor solids concentration is considerably reduced. The settler does become slightly underloaded during a portion of the cycle, and some deterioration in performance is evident. The situation is prevented from becoming serious here by the cyclic nature of the process, resulting in a sharp increase in substrate feed and a consequent increase in solids growth rate during the period in which underloading causes a decrease in the recycled solids concentration. A higher initial sludge height would, of course, have prevented underloading and resulted in an ideal control situation, while an initially lower sludge height would have shown even poorer response under ratio control. As shown in Figure 7, there is a negative sludge waste rate over more than 40% of the cycle, so sludge storage would be required.

RECYCLE AND UNDERFLOW RATIO CONTROL

There are two important considerations in control of the activated sludge process: the settler must be kept from becoming underloaded or overloaded and large fluctuations in the reactor solids concentration must be prevented. Ratio control on the recycle satisfies the second of these requirements, but only with sludge storage, and large variations in sludge height may occur.

The sludge height can be controlled by changing the settler underflow rate. At steady state there is a single underflow rate for given reactor feed conditions and a given recycle rate for which the settler is neither underloaded nor overloaded. This underflow rate can be computed for each feed level shown in Figure 5, assuming ratio control on the recycle, and over the entire range of feed fluctuations shown, the ratio of underflow to feed rate is constant to within 1%. Thus, in a pseudo-steady state environment, a ratio control on the settler underflow rate would maintain the sludge height at a constant position. A constant level would not be expected under dynamic conditions, but underflow ratio control is clearly suggested.

Simulation results for the disturbance time series in Fig. 4 are shown as line III in Figures 6 and 7 and in Figure 8. Both the recycle rate and underflow rate are maintained at the constant design ratio relative to the feed flow rate. The sludge blanket height varies little over a 24 hr cycle, and neither underloading nor overloading presents any danger. There are large variations in underflow concentration, and there is some variation in reactor solids concentration, but the reactor fluctuations are reduced to 30% of those in the system without control. The rate of sludge wastage can never be negative with this control policy, so sludge storage is not required.

The total waste solids and mean waste solids concentration are nearly the same for constant underflow and recycle and ratio control on underflow and recycle. Thus, the possible undesirable increase in sludge volume suggested by Olsson (1976) when ratio recycle control and sludge blanket height control are both implemented does not occur in this case.

The concentration of substrate in the effluent is small in all cases and does not vary significantly among the three operating policies tested. The possible effect of different settler loadings on suspended solids in the effluent is not accounted for in the model, however.

CONCLUSION

The use of an activated sludge model that incorporates settler dynamics is necessary in order to account properly for interactions between the settler and the reactor. Settler underloading can lead to extremely sluggish system response, and one objective of process control should be maintenance of a steady height of the sludge blanket. The simulations reported here show that this objective is sometimes in conflict with a second goal, maintenance of a constant solids concentration in the reactor.

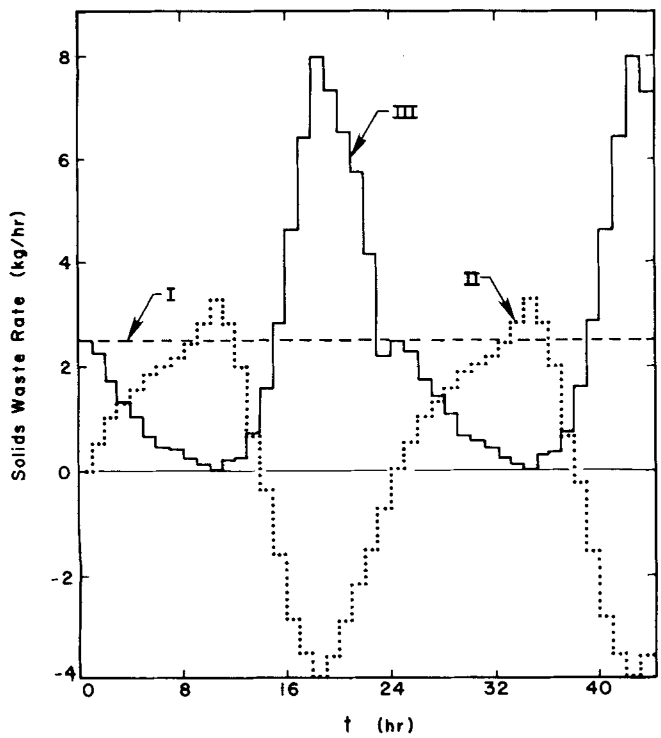

Fig. 7. Rate of solids wasting for input forcing in Figure 5. I, II, and III have the same meaning as in Figure 6.

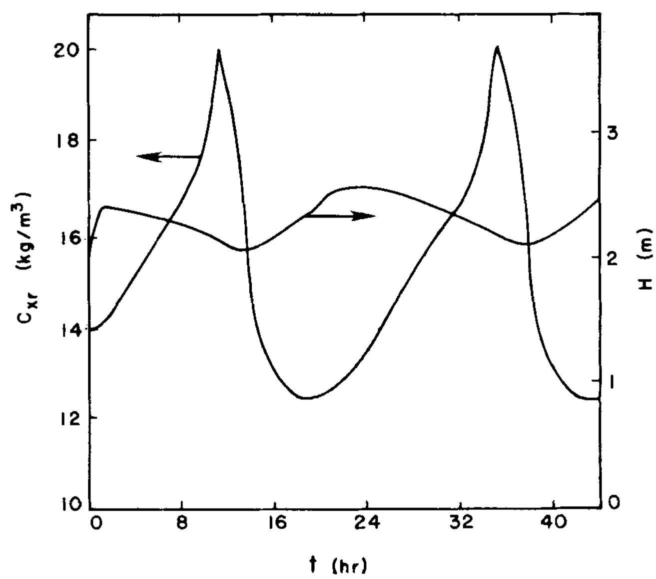

Fig. 8. Sludge height and settler underflow concentration for input forcing in Figure 5 with ratio control on recycle and underflow.

An effective compromise between these two objectives can be achieved by using ratio control on both the recycle and underflow streams. No sludge storage is required, variations in the sludge height are slight, and fluctuations in the reactor solids concentration are small relative to the uncontrolled case.

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NOTATION

C = concentration
G = solids flux
h = height of a layer in settler
H = height of sludge blanket
k = constant in reaction rate
Q = volumetric flow rate
r = volumetric recycle ratio from settler
R = reaction rate
t = time
u = underflow velocity from settler
v = solids settling velocity in settler
V = reactor volume
w = sludge volumetric wasting rate
Y = yield factor
δ = velocity of solids discontinuity in settler
μ = substrate dependent portion of reaction rate
$\hat{\mu}$ = coefficient in reaction rate
τ = time constant for reaction rate

Subscripts

f = feed
n = layer number in settler
r = recycle
s = substrate (BOD)
x = solids

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Countercurrent Backmixing Model for Slugging Fluidized-Bed Reactors

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A model is developed for slugging fluidized bed catalytic reaction which improves on the model of Hovmand and Davidson by incorporating the solids mixing. The model parallels the countercurrent backmixing model of Fryer and Potter. Solids mixing is accounted for by the model of Thiel and Potter which assumes the rising slug is followed by a well-mixed wake into and out of which solids flow at a rate determined by the rise velocity of the slug. Comparison is made with experimental data reported by Hovmand and Davidson with good results.

SCOPE

The design of commercial size, gas fluidized-bed reactors is generally based on data obtained from reactors of the laboratory or the pilot plant scale. In these smaller reactors, slug flow often occurs owing to rapid coalescence of the bubbles above the distributor plate, particularly when the height of the bed is large compared to its diameter. Therefore, it is necessary to understand the effects of slug flow on conversion, so that laboratory or pilot plant data may be used for scaling-up.

Hovmand and Davidson (1971) proposed a two-phase model for slugging beds, similar to the two-phase model for bubbling beds by Davidson and Harrison (1963). Com-